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13. ABSTRACT *(Maximum 200 words)*

The main focus of this research was determining the accuracy of subspace based methods for estimating the Direction of Arrival (DOA) of multiple sources from measurements obtained at the output of a sensor array. Subspace methods like MUSIC (MUltiple SIgnal Classification), ESPRIT (Estimation of Signal Parameters via Rotational Invariant Techniques), the Minimum-Norm methods have recently received much attention, and their estimation accuracy as well as a rigorous comparative study is of much interest. This was the goal of this research. Of particular interest was the affect of spatial smoothing on the performance

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of the subspace methods. Spatial smoothing is useful in dealing with coherent sources and for the possible enhancement of the performance of the methods. Also, examined were the implementation issues associated with these methods. As opposed to implementing a single algorithm, implementing a signal processing task which consists of several stages on special purpose hardware gives prominence to the interesting issues of partitioning, and composite tasking, which are examined in this research. The results have significantly improved the understanding of the performance of subspace methods, and have lead to interesting insights into the implementation issues.

FINAL REPORT

TITLE: PERFORMANCE ANALYSIS OF SUBSPACE METHODS

AUTHOR: BHASKAR D. RAO

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FINAL REPORT

This research was funded from April 90 - March 92, with a no cost extension granted till March 1993. This report summarizes the research results that were obtained primarily during the period April 90 - March 92. Only the main results are summarized, and for the details the reader is referred to our publications. Also included at the end of the report are the abstracts of the publications.

1 Problem Statement

The main focus of this research was determining the accuracy of subspace based methods for estimating the Direction of Arrival (DOA) of multiple sources from measurements obtained at the output of a sensor array. Subspace methods like MUSIC (MUltiple SIgnal Classification), ESPRIT (Estimation of Signal Parameters via Rotational Invariant Techniques), the Minimum-Norm methods have recently received much attention, and their estimation accuracy as well as a rigorous comparative study is of much interest. This was the goal of this research. Of particular interest was the affect of spatial smoothing on the performance of the subspace methods. Spatial smoothing is useful in dealing with coherent sources and for the possible enhancement of the performance of the methods. Also, examined were the implementation issues associated with these methods. As opposed to implementing a single algorithm, implementing a signal processing task which consists of several stages on special purpose hardware gives prominence to the interesting issues of partitioning, and composite tasking, which are examined in this research. We believe our results have significantly improved the understanding of the performance of subspace methods, and have lead to interesting insights into the implementation issues.

2 Main results

Our main results in the context of performance analysis of spatially smoothed subspace based methods are as follows.

- We have established a general framework for the analysis of subspace methods. The usefulness of the framework is that it is simple inspite of its generality. This makes it

possible to analyze the more complicated problem of performance analysis of spatially smoothed subspace methods. Past work had mainly considered subspace methods in the context of no smoothing.

- We have shown that the forward-backward covariance matrix is better conditioned than the forward only covariance. This provides a firmer mathematical basis for the previous experimental observation that forward-backward approaches have better performance than forward only approaches.

- Expression for the mean squared error in the estimates obtained using the subspace based methods have been derived and compared. Various properties of the estimators are derived based on these expressions. It is shown that spatial smoothing is beneficial to ESPRIT and the Minimum-Norm method, and that their performance can be made comparable to MUSIC by proper smoothing.

- Weighted versions of these subspace methods have also been examined and optimal choices for the weights are determined. An interesting conclusion is that the performance of optimally weighted MUSIC and optimally weighted ESPRIT are the same.

- Another interesting outcome of the analysis is that it is shown that the analysis can be easily applied to the time series frequency estimation problem. In addition to providing a unified framework, the analysis is more accurate than previous analysis.

In the context of implementation issues, the problem of partitioning becomes an important one when implementing a signal processing task that includes a number of stages. It is examined in great detail, and new scheduling methods for the Locally Parallel Globally Sequential (LPGS) technique and the Locally Sequential Globally Parallel (LSGP) technique are developed. A flexible scheduling order is developed that is useful in evaluating the trade-off between execution times and the size of storage buffers. The results are revealing and lead to many interesting insights. For example, while comparing Givens method with Housholder Transformation (HT) for Least Squares problem, it is shown that HT is superior when the matrices are large compared to the array size.

3 Publications

These publications were supported by the U. S. Army Research Office under Grant Number DAAL-03-90-G-0095.

Time Period: April 90 - March 93.

1. B. D. Rao and K.V.S. Hari, "Spatial Smoothing and MUSIC: Further Results," pp. 261-276, SVD and Signal Processing, II, Algorithms, Analysis and Implementation, Edited by R. J. Vaccaro, Elsevier Science Publishers B.V., 1991. Also appeared in the 2nd International Workshop on SVD and Signal Processing, June 25-27, 1990.

2. B. D. Rao and K. V. S. Hari, "Effect of Spatial Smoothing on State Space Methods/ESPRIT," Fifth ASSP Workshop on Spectrum Estimation and Modeling, pp. 377-381, Oct. 10-12, Rochester, New York, 1990.

3. B. D. Rao and K. V. S. Hari, "On Spatial Smoothing and Weighted Subspace Methods," pp. 936-940, Twenty-Fourth Asilomar Conference on Signals, Systems and Computers, Nov. 5-7, Monterey, California, 1990.

4. B. D. Rao, "A Systematic Approach for the Analysis of Roundoff Noise in Floating Point Digital Filters," pp. 495-499, Twenty-Fourth Asilomar Conference on Signals, Systems and Computers, Nov. 5-7, Monterey, California, 1990.

5. B. D. Rao, and K.V.S. Hari, "Effect of Spatial Smoothing on the Performance of MUSIC and the Minimum-Norm Method," IEE Part F: Radar and Signal Processing, pp. 449-458, Dec. 1990.

6. B. D. Rao and K. V. S. Hari, "Weighted State Space Methods/ESPRIT and Spatial Smoothing," pp. 3317-3320, Vol. 5, Proc. of the International Conference on Acoustics, Speech and Signal Processing, Toronto, Canada, May 14-17, 1991.

7. B. D. Rao, "Analysis of Roundoff Noise in Floating Point Digital Filters," pp. 1893-1896, Vol. 2, Proc. of the International Conference on Acoustics, Speech and Signal Processing, Toronto, Canada, May 14-17, 1991.

8. B. D. Rao and K.V.S. Hari, "Analysis of Subspace Based Direction Of Arrival Estimation Methods," Sadhana - A proceedings of Indian Academy of Sciences in Engineering

Sciences - Special Issue on Recent Advances in Digital Signal Processing - II, Vol. 16, Part 3, pp. 183-194, November 1991.

9. P. Kuchibhotla and B. D. Rao, "Partitioning considerations in Systolic Array Design," pp. 530-534, Vol. 1, Twenty-Fifth Asilomar Conference on Signals, Systems and Computers, Monterey, California, Nov. 1991.

10. B. D. Rao, "Comparison of Spatially Smoothed Weighted Subspace Methods," pp. 903-907, Vol. 2, Twenty-Fifth Asilomar Conference on Signals, Systems and Computers, Monterey, California, Nov. 1991.

11. B. D. Rao, "Floating Point Arithmetic and Digital Filters," IEEE Trans. on Signal Processing, pp. 85-95, Vol. 40, No. 1, Jan. 1992.

12. B. D. Rao, and K.S. Arun, "Model Based Processing of Signals: A State Space Approach," Proc. of IEEE, pp. 283-309, February 1992.

13. P. Kuchibhotla and B. D. Rao, "Efficient scheduling methods for partitioned systolic algorithms," International Conference on Application Specific Array Processors, August 4-7, Berkeley, CA, 1992.

14. P. Kuchibhotla and B. D. Rao, "Scheduling parallel implementations of partitioned orthogonal transformations," SPIE's Annual conference, July 19-24, 1992, San Diego, California.

15. I. F. Gorodnitsky and B. D. Rao, "A new iterative weighted norm minimization algorithm and its applications," Sixth SP workshop on statistical signal and array processing, Oct. 7-9, 1992, Victoria, B.C. Canada.

16. B. D. Rao and K. V. S. Hari, "Weighted Subspace Methods and Spatial Smoothing: Analysis and Comparison" IEEE Trans. on Signal Processing, pp. 788-803, Vol. 41, No. 2, February 1993.

17. P. Kuchibhotla and B. D. Rao, "A methodology for fast scheduling of partitioned systolic algorithms," submitted to the Journal of VLSI Signal Processing.

18. P. Kuchibhotla and B. D. Rao, "An index prediction based Vector Quantization scheme," submitted to IEEE Trans. on Image Processing

4 Personnel

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- Prashanth Kuchibhotla (Research Assistant)
- Irina Gorodnitsky (Research Assistant)

5 Paper Abstracts

Attached are the abstracts of papers published during this period and supported by the grant.

SVD AND SIGNAL PROCESSING, II
Algorithms, Analysis and Applications
R.J. Vaccaro (Editor)

Spatial Smoothing and MUSIC : Further Results

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Abstract

In this paper, new results concerning the effect of using a spatially smoothed forward-backward covariance matrix on the performance of MUSIC for the direction of arrival (DOA) estimation using a uniformly spaced linear sensor array (ULA) are presented. Compact expressions for the asymptotic mean squared error in the estimates of the signal zeros and the DOA estimates are derived. Some general properties of the estimates are derived which lend insight into the effect of spatial smoothing and the forward-backward approach on MUSIC. An optimally *weighted* MUSIC algorithm is also presented which minimizes the mean squared error in the DOA estimate.

1. INTRODUCTION

In this paper, new results concerning the effect of using a spatially smoothed forward-backward covariance matrix on the statistical performance of MUSIC [1] are presented. In recent years, a statistical evaluation of MUSIC has been conducted by a number of researchers [2,3,4,5,6]. In all these studies, an estimate of the covariance matrix obtained by straight forward averaging of the outer product of the snapshots was considered. Such a covariance estimate is not suitable for coherent environments. As a solution to the problem that arises with coherent sources, the idea of spatial smoothing was suggested [7,8]. In this paper, we examine the effect of using data both in the forward and backward manner along with spatial smoothing on the performance of MUSIC. An analysis of MUSIC under these conditions has been presented in [9,10,11]. The results in this paper extend these results and provide more tractable expressions which provide additional insight into MUSIC. Compact expressions for the mean squared error (MSE) in the signal zeros as well as in the DOA estimate are derived, and these results provide a better understanding of Root and Spectral forms. It is found that for MUSIC, the error in the signal zeros follows a different trend compared to the error in the DOA estimates leading to difficulty in interpreting the spatial spectrum. Some new relationships between the DOA estimates

Effect of Spatial Smoothing on State Space Methods/ESPRIT*

Bhaskar D. Rao and K. V. S. Hari
University of California, San Diego

ABSTRACT

In this paper, results concerning the effect of using a spatially smoothed forward-backward covariance matrix on the performance of State Space Methods/ESPRIT for the direction of arrival (DOA) estimation using a uniformly spaced linear sensor array (ULA) are presented. Compact expressions for the asymptotic mean squared error in the estimates of the signal zeros and the DOA estimates are derived. Some general properties of the estimates are derived which lend insight into the effect of spatial smoothing and the forward-backward approach on State Space Methods/ESPRIT. An optimally *weighted* State Space Method/ESPRIT algorithm is also presented. The results indicate that by properly choosing the number of subarrays used for spatial smoothing, the performance of the method can be greatly enhanced.

1 INTRODUCTION

Recently a statistical evaluation of eigendecomposition based State Space Methods/ESPRIT was conducted when an estimate of the covariance matrix was obtained by straight forward averaging of the outer product of the snapshots [1,2]. Such a covariance estimate is not suitable for coherent environments. As a solution to the problem that arises with coherent sources, the idea of spatial smoothing was suggested [3,4]. Also using data both in the forward and backward manner along with spatial smoothing can improve the quality of the covariance estimate thereby improving the DOA estimate. In this paper, we analytically quantify the effect of such covariance estimates on the DOA estimates obtained using State Space Methods/ESPRIT. Compact expressions for the mean squared error (MSE) in the signal zeros as well as in the DOA estimate are derived. Some new relationships between the DOA estimates obtained from the various covariance estimates are presented. Based on the insight gained from the analysis, an optimally *weighted* State Space Method/ESPRIT is also developed which minimizes the MSE in the DOA estimates. A more detailed treatment of this subject can be found in [5], and results related to spatial smoothing and MUSIC can be found in [6].

*This work was supported by the ARMY Research Office under Grant No. DAAL-03-90-G-0095.

2 PROBLEM FORMULATION

2.1 Data Model

The problem of estimating the direction of arrival of M possibly coherent plane waves incident on a ULA of L_1 sensors is considered in this paper. For the n th observation period (snapshot), the spatial samples of the signal plus noise are given by the output vector

$$Y(n) = [\, y_1(n)\ ,\ y_2(n)\ ,\ \ldots\ ,\ y_{L_1}(n)\ ]^T = X(n) + N(n).$$

where

$$x_k(n) = \sum_{i=1}^{M} p_i(n)\ e^{j(k-1)\omega_i}$$

and $\omega_i = \dfrac{2\pi d}{\lambda} \sin\theta_i$, d being the separation between sensors, λ the wavelength of the incident signal, and θ_i the DOA. The following assumptions are made regarding the data [1].

A1 The noise vector $N(n)$ is assumed to be a zero mean, complex, white circularly Gaussian random vector, i.e. $\overline{N(n)\ N(n)^H} = \sigma_n^2\ I$ and $\overline{N(n)\ N(n)^T} = 0$. It is also assumed to be independent of the complex signal amplitudes $p_i(n)$.

The exact distribution of the source amplitudes $p_i(n)$ and hence that of $X(n)$ turns out to be not so important in characterizing the asymptotic behavior [5,7]. However, for simplicity, in this paper the following assumption will be made.

A2 The complex amplitudes $p_i(n)$ are also modeled as being zero mean circularly Gaussian random variables.

The covariance matrix of the amplitudes is P whose elements are P_{ij}, where $P_{ij} = \overline{p_i(n)\, p_j^*(n)}$.

[1] In this paper, the overbar "–" will be used to denote the expectation operator. The superscript T is used to denote transpose, * to denote complex conjugate, H to denote complex conjugate transpose, and + to denote the pseudoinverse.

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ON SPATIAL SMOOTHING AND WEIGHTED SUBSPACE METHODS*

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Abstract

The statistical performance analysis of *Weighted* MUSIC and Weighted State Space Methods/ESPRIT, for the direction of arrival (DOA) estimation problem using a uniform linear array is presented in this paper. The statistics of the errors in the subspaces due to the forward-backward spatially smoothed covariance matrix estimate is presented, and its effect on the weighted subspace methods is discussed. Based on the analysis, an optimum weighting matrix minimizing the mean squared error in one of the DOA is derived. It is shown that optimum weighting improves the performance of the subspace methods and makes them less sensitive to the choice of the sub-array length.

1 Introduction

In this paper, results concerning the effect of using a spatially smoothed forward-backward covariance matrix on the statistical performance of Weighted subspace methods, weighted MUSIC and weighted State Space Methods/ESPRIT, are presented. Extending the analysis of unweighted subspace methods, compact expressions for the mean squared error (MSE) in the signal zeros as well as in the DOA estimate are derived. Based on the insight gained from the analysis, an optimal *weighting* matrix which minimizes the MSE in a given DOA is obtained.

2 Problem Formulation

2.1 Data Model

For the n th observation period (snapshot), the spatial samples when M possibly coherent plane waves are incident on a Uniformly spaced Linear Array (ULA) of L_1 sensors are given by the output vector

$$\begin{aligned} Y(n) &= [\, y_1(n)\,,\, y_2(n)\,,\,\ldots\,,\, y_{L_1}(n)\,]^T \\ &= \left[\sum_{i=1}^{M} p_i(n), \ldots, \sum_{i=1}^{M} p_i(n) e^{j(L_1-1)\omega_i}\right]^T + N(n) \\ &= X(n) + N(n), \end{aligned}$$

where $\omega_i = \dfrac{2\pi d}{\lambda}\sin\theta_i$, d being the separation between sensors, λ the wavelength of the incident signal, and θ_i the DOA. The following assumptions are made regarding the data[1].

- **A1** The noise vector $N(n)$ is assumed to be a zero mean, complex, white circularly Gaussian random vector, i.e. $\overline{N(n)\,N(n)^H} = \sigma_n^2\, I$, $\overline{N(n)\,N(n)^T} = 0$ and is independent of the complex signal amplitudes $p_i(n)$.
- **A2** The complex amplitudes $p_i(n)$ are modeled as zero mean circularly Gaussian random variables.
- **A3** The N snapshot vectors are assumed to be independent.

The covariance matrix of the amplitudes is P whose elements are P_{ij}, where $P_{ij} = \overline{[p_i(n)\,p_j^*(n)]}$. Though, the Gaussian assumption on the signal amplitudes (A2) is not essential for the analysis, it is assumed in this paper for simplicity.

2.2 Different Covariance Matrix Estimators

Some of the popular estimators are defined below

Forward only (F) approach : In this approach, an estimate of the covariance matrix is obtained by straightforward (time) averaging of the outer product of the N snapshots[2]

$$\hat{R}_f = \frac{1}{N}\sum_{n=1}^{N} Y(n)\, Y^H(n)$$

Forward only Smoothing (FS) approach :

$$\hat{R}_{fs} = \frac{1}{K}\sum_{p=1}^{K} \hat{R}_p^{fs} \;\; where \;\; \hat{R}_p^{fs} = \frac{1}{N}\sum_{n=1}^{N} Y_p(n)\, Y_p^H(n),$$

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[1] In this paper, the overbar "‾" will be used to denote the expectation operator. The superscript T is used to denote transpose, * to denote complex conjugate, H to denote complex conjugate transpose, and + to denote the pseudoinverse.

[2] ˆ is used to denote estimates

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A SYSTEMATIC APPROACH FOR THE ANALYSIS OF ROUNDOFF NOISE IN FLOATING POINT DIGITAL FILTERS*

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Abstract

In this paper, a systematic approach for the analysis of roundoff noise in floating point digital filters is presented. To make the analysis tractable, a high level model to deal with the errors in the inner product operation is presented. Along with the model, an efficient procedure to keep track of the addition scheme used in the inner product, and to compute the statistics of the errors is introduced. A systematic procedure based on the model is then developed to derive general expressions for the roundoff noise of FIR and IIR filters.

1 INTRODUCTION

With the increasing availability of floating point capability in signal processing chips, insight into algorithms employing floating point arithmetic is of increasing interest. This paper develops a systematic and tractable procedure for analyzing roundoff noise in digital filters employing floating point arithmetic. The related problem of using fixed point arithmetic and their effect on digital filters has been extensively studied and is reasonably well understood [1,2,3,4,5]. The analysis of floating point digital filters has also been studied by researchers [6]-[11]. Unfortunately, the expressions turn out to be cumbersome and not as insightful. Part of the reason for the difficulty lies in the model for finite precision floating point arithmetic. Unlike the case of fixed point arithmetic, where the roundoff errors could be modeled as a independent white noise sequences independent of the input signal, the roundoff errors in floating point arithmetic are correlated with the input signal complicating the analysis [7,6]. Also, unlike the fixed point case, the order of the computations has an effect on the roundoff error. Recently in [12], it was shown that use of the Factored State Variable Description can greatly enhance the tractability of the analysis. Also in [13,14], a procedure for analyzing roundoff noise based on coefficient sensitivity analysis is presented.

This paper examines the issue of output roundoff noise of floating point digital filters with a view towards generalizing previous results [15,7,6]. An effective way for dealing with floating point inner product operations is presented. In particular, an explicit procedure for dealing with the effect of addition schemes is provided. Based on these results, a systematic and tractable approach for analyzing roundoff noise in floating point digital filters is presented. The results of this paper are an extension of our earlier work [16].

2 ERRORS IN FLOATING POINT ARITHMETIC

Throughout this paper it will be assumed that floating point numbers are stored in the form $(sign).\mu.2^{\nu}$, where μ and ν have a fixed number of bits. Also it will be assumed that rounding is used in all operations. The notation $fl(.)$ will be used to denote the machine number resulting from floating point operation. In floating point operations there are errors in both additions and multiplications [7,6], i.e.

$$fl(x + y) = (x + y)(1 + \epsilon), \tag{1}$$

and

$$fl(xy) = xy(1 + \delta), \tag{2}$$

and $-2^{-q} \leq \epsilon, \delta \leq 2^{-q}$, q being the number of bits used to represent the mantissa. The error variables ϵ and δ are usually assumed to be random variables uniformly distributed between -2^{-q} and 2^{-q}. The error variables have zero mean and variance $\sigma_r^2 = \frac{2^{-2q}}{3}$. Sometimes a slightly different distribution is assumed leading to a variance $\sigma_r^2 = \frac{2^{-2q}}{8\ln 2}$. For the purposes of this paper, this is not critical as it simply scales the overall noise expressions derived. The above basic/elementary model of the

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Effect of spatial smoothing on the performance of MUSIC and the minimum-norm method

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Indexing terms: Algorithms, Signal processing, Direction finding

Abstract: The effect of using a spatially smoothed forward–backward covariance matrix on the performance of MUSIC and the minimum-norm method for estimating the direction of arrival of plane waves in white noise using a uniformly spaced linear array (ULA) is analysed. In particular, asymptotic results for the mean squared error in the estimates of the signal zeros and the direction of arrival are derived. It is shown that in general for subspace methods a forward–backward smoothing approach is preferable to a forward smoothing approach. An important outcome of this analysis is that for MUSIC, the error in the signal zeros is shown to exhibit a different trend compared to the error in the DOA estimates and this leads to difficulty in interpreting the spatial spectrum. For instance, when smoothing is used, the peaks in the spatial spectrum become sharper, giving the impression of higher resolution, whereas in reality the estimates of the DOA may in fact have deteriorated compared with the ones obtained using minimal or no smoothing. With regard to the minimum-norm method, the errors in the signal zeros exhibit the same trend as the DOA estimates so that no such problem is created. As to the relative comparison of the methods, it is shown that proper spatial smoothing enables the performance of the minimum-norm method to be made comparable to MUSIC.

1 Introduction

In this paper, we analyse the effect of using a spatially smoothed forward–backward covariance matrix on the statistical performance of two eigendecomposition-based methods, MUSIC [1] and the minimum-norm method [2]. For better understanding, a statistical evaluation of these methods has been conducted in recent years by a number of researchers. For instance, some theoretical results comparing MUSIC and the minimum-norm method can be found in References 3 and 4, wherein the methods were characterised by examining the null spectrum. More recent work on the analysis of MUSIC can be found in References 5–7. Our recent work examined root-MUSIC and root-minimum-norm in detail for the uniformly spaced linear array (ULA) case [7, 8]. In all these studies, an estimate of the covariance matrix obtained by straightforward averaging of the outer product of the snapshots was considered. Such an approach is not suitable for coherent environments. As a solution to this problem, the idea of spatial smoothing has been suggested [9, 10]. Here we examine the effect of using data both in the forward and backward manner, along with spatial smoothing, on the performance of these methods. An analysis of the methods under these conditions has been presented in References 11–13. The results in this paper extend results of these references and provide additional insight into the methods. Motivated by our earlier results [7, 8], the root versions of these methods are examined in this paper. Expressions for the mean squared error in the signal zeros as well as in the direction-of-arrival (DOA) estimate are determined and these results provide insight into both root and spectral forms. It is shown that, in general, for all subspace methods the forward–backward smoothing approach is preferable to the forward smoothing approach. The error in the signal zeros obtained using MUSIC is shown to follow a different trend compared to the error in the DOA estimates, and this leads to difficulty in interpreting the spatial spectrum. On the other hand, it is shown that for the minimum-norm method the errors in the signal zeros exhibit the same trend as the DOA estimates so that no such problem is created. Also it is shown that proper spatial smoothing enables the performance of the minimum-norm method to be made comparable to that of MUSIC. Some of these results were first presented in Reference 14 and results relating to spatial smoothing and state space methods/ESPRIT can be found in Reference 15.

Paper 7581F (E5, E11, E15), first received 7th December 1989 and in revised form 6th August 1990

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2 Problem formulation

2.1 Data model

The performance analysis of subspace methods used for estimating the directions of arrival of M possibly coherent plane waves incident on a ULA of L_1 sensors is considered in this paper. For the nth observation period (snapshot), the spatial samples of the signal plus noise are given by the output vector

$$\begin{aligned} Y(n) &= [y_1(n), y_2(n), \ldots, y_{L_1}(n)]^T \\ &= \left[\sum_{i=1}^{M} p_i(n), \sum_{i=1}^{M} p_i(n) e^{j\omega_i}, \ldots, \sum_{i=1}^{M} p_i(n) e^{j(L_1-1)\omega_i} \right]^T \\ &\quad + N(n) \\ &= X(n) + N(n) \end{aligned}$$

where $\omega_i = (2\pi d/\lambda) \sin \theta_i$, d being the separation between sensors, λ the wavelength of the incident signal, and θ_i

E5.14

Weighted State Space Methods/ESPRIT and Spatial Smoothing*

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Abstract

In this paper, the effect of spatial smoothing on direction of arrival (DOA) estimates obtained using weighted eigen-based State Space methods/ESPRIT is analyzed. Expressions for the asymptotic mean squared error in the estimates of the signal zeros, and the DOA estimates along with some general properties of the estimates are derived. It is shown that proper choice of the subarray length can improve the performance of the method significantly. Based on the asymptotic expressions, an optimum weighting matrix minimizing the mean squared error in one of the DOA is derived.

1 INTRODUCTION

This paper considers the effect of spatial smoothing on direction of arrival (DOA) estimates obtained using weighted eigen-based state space methods/ESPRIT in the context of Uniformly spaced Linear Arrays (ULA). Analysis of the method for the forward only (no smoothing) case has been conducted by a number of researchers [1,2,3]. In [2,4,1], it was shown that MUSIC performs better than eigen-based state space methods with the difference becoming significant as the length of the array increases. However, state space methods are computationally more attractive, and it is desirable to see if this deficiency can be overcome by using a spatially smoothed forward-backward covariance matrix. This is shown to be the case in this paper. It is shown that proper choice of subarray length can significantly improve the performance of the method. Also it is shown that instead of using a simple least squares method, if an optimally weighted least squares procedure is used in the estimation of the state transition matrix, the performance of the method is enhanced with respect to a DOA, and the sensitivity to the choice of the subarray length is reduced. A more detailed treatment of these issues can found in [5,6,7].

*This work was supported by the ARMY Research Office under Grant No. DAAL-03-90-G-0095

2 Problem Formulation

2.1 Data Model

For the n th observation period (snapshot), the spatial samples when M possibly coherent plane waves are incident on a ULA of L_1 sensors are given by the output vector

$$\begin{aligned} Y(n) &= \left[\sum_{i=1}^{M} p_i(n), \ldots, \sum_{i=1}^{M} p_i(n) e^{j(L_1-1)\omega_i}\right]^T + N(n) \\ &= X(n) + N(n). \end{aligned}$$

where $\omega_i = \frac{2\pi d}{\lambda}\sin\theta_i$, d being the separation between sensors, λ the wavelength of the incident signal, and θ_i the DOA. The following assumptions are made regarding the data[1].

A1 The noise vector $N(n)$ is assumed to be a zero mean, complex, white circularly Gaussian random vector, i.e. $\overline{N(n)\,N(n)^H} = \sigma_n^2 I$, $\overline{N(n)\,N(n)^T} = 0$ and is independent of the complex signal amplitudes $p_i(n)$.

A2 The complex amplitudes $p_i(n)$ are modeled as zero mean circularly Gaussian random variables.

A3 The N snapshot vectors are assumed to be independent.

The covariance matrix of the amplitudes is P whose elements are P_{ij}, where $P_{ij} = \overline{[p_i(n)\,p_j^*(n)]}$. Though, the Gaussian assumption on the signal amplitudes (A2) is not essential for the analysis [6,3], it is assumed in this paper for simplicity.

2.2 Different Covariance Matrix Estimators

Forward only (F) approach : In this approach, an estimate of the covariance matrix is obtained by straightforward (time) averaging of the outer product of the N

[1] In this paper, the overbar "‾" will be used to denote the expectation operator. The superscript T is used to denote transpose, * to denote complex conjugate, H to denote complex conjugate transpose, and + to denote the pseudoinverse.

D6.4

ANALYSIS OF ROUNDOFF NOISE IN FLOATING POINT DIGITAL FILTERS*

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Abstract

In this paper, a systematic approach for the analysis of roundoff noise in floating point digital filters is presented. The analysis is based on a high level model developed to deal with the errors in the inner product operation. The model consists of an efficient procedure to keep track of the addition scheme used in the inner product, and to compute the statistics of the errors. The tractability of the analysis is demonstrated by deriving general expressions for the roundoff noise of FIR and IIR filters. Also the connection between coefficient sensitivity analysis and roundoff noise analysis are discussed.

1 INTRODUCTION

This paper develops a systematic and tractable procedure for analyzing roundoff noise in digital filters employing floating point arithmetic. The related problem of using fixed point arithmetic and their effect on digital filters has been extensively studied and is reasonably well understood [1,2]. The analysis of floating point digital filters has also been studied by researchers [3,4,5]. Unfortunately, the expressions turn out to be cumbersome and not as insightful. Part of the reason for the difficulty lies in the model for finite precision floating point arithmetic. Unlike the case of fixed point arithmetic, where the roundoff errors could be modeled as a independent white noise sequences independent of the input signal, the roundoff errors in floating point arithmetic are correlated with the input signal complicating the analysis [4,3]. Also, unlike the fixed point case, the order of the computations has an effect on the roundoff error. Recently in [6], it was shown that use of the Factored State Variable Description can greatly enhance the tractability of the analysis. Also in [7], a procedure for analyzing roundoff noise based on coefficient sensitivity analysis is presented.

This paper examines the issue of output roundoff noise of floating point digital filters with a view towards generalizing previous results [8,4,3]. The results of this paper are an extension of our earlier work [9], and more details can be found in [10].

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2 ERRORS IN FLOATING POINT ARITHMETIC

In this paper, it will be assumed that rounding is used in all operations.The notation $fl(.)$ will be used to denote the machine number resulting from floating point operation. In floating point operations there are errors in both additions and multiplications [4,3], i.e.

$$fl(x+y) = (x+y)(1+\epsilon), \tag{1}$$

and

$$fl(xy) = xy(1+\delta), \tag{2}$$

and $-2^{-q} \leq \epsilon, \delta \leq 2^{-q}$, q being the number of bits used to represent the mantissa. The error variables ϵ and δ are assumed to have zero mean and variance σ_r^2 (usually $\sigma_r^2 = \frac{2^{-2q}}{3}$ is assumed). The above basic/elementary model of the errors has been used by a number of researchers to study floating point digital filters.

Note that the errors, depend on the variables involved in the operation. In the context of floating point digital filters, this amounts to dependence of the output roundoff error on the input signal and the intermediate variables generated in the filter making the analysis cumbersome and less tractable. This is particularly true when the number of operations is large. Here we seek some tools for the purpose of enhancing the tractability of the analysis and thereby hopefully lead to new insight and results.

3 DIGITAL FILTERS AND FLOATING POINT INNER PRODUCTS

3.1 Inner Products and Digital Filters

In filter implementation, instead of starting with the basic error model it is useful to recognize that inner products

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Analysis of subspace-based direction of arrival estimation methods

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Abstract. In this paper, a general framework for the analysis of eigen-based subspace methods is developed. It is shown that a two-step procedure can be effectively used to analyse subspace methods under fairly general conditions. The first step relates the errors in the covariance matrix to errors in the subspaces, and the second step relates error in the subspaces to the errors in the direction of arrival (DOA) estimates. Combining these two steps along with the statistics of the data, expressions for the mean squared error in the DOA estimate are derived. The potential of the approach is demonstrated by analysing two subspace methods, MUSIC and the minimum-norm method.

Keywords. Array processing; direction of arrival estimation; subspace methods; statistical analysis.

1. Introduction

Eigen-decomposition based methods have recently received a great deal of attention (Bienvenu & Kopp 1979; Schmidt 1979; Kumaresan & Tufts 1983; Cadzow 1988; Rao 1988; Ottersten 1989; Roy & Kailath 1989; Viberg 1989). These methods, referred to as subspace methods, decompose the space spanned by the eigenvectors of the observed covariance matrix into two orthogonal subspaces referred to as the signal and noise subspace. In the recent past, an extensive analysis of these methods has been conducted by a number of researchers providing valuable insight. The first work in this context has been presented in Jeffries & Farrier (1985) and Kaveh & Barabell (1986), where the resolution capability of MUSIC and the Minimum-Norm method were studied by examining the null spectrum. Recently, expressions for the asymptotic mean squared error in the direction of arrival (DOA) estimate have been developed (Porat & Friedlander 1988; Rao & Hari 1989a, b; Stoica & Nehorai 1989, 1991). The results have also been extended to deal with the effect of spatial smoothing (Clergeot *et al* 1989; Pillai & Kwon 1989; Rao & Hari 1990) and sensor errors (Friedlander 1989; Swindlehurst *et al* 1989). Using the insight gained from these results, a general framework for analysing subspace methods is developed. The overall

Partitioning Considerations in systolic array design *

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Abstract

Various methods for mapping algorithms into systolic arrays have been developed in the past few years. In this paper, efficient scheduling techniques are developed for methods in partitioning problems that do not match the array size exactly. In particular, scheduling for the Locally Parallel-Globally Sequential (LPGS) technique and the Locally Sequential-Globally Parallel (LSGP) technique are developed. The scheduling procedure developed exploits the fact that after LPGS and LSGP partitioning, the locality constraints get modified allowing one to use certain inter-connections that were not available before. The scheduling method allows us to develop a flexible scheduling order for LPGS that is useful in evaluating a trade-off between execution time and size of partitioning buffers. The scheduling techniques are illustrated with the help of matrix multiplication and QR decomposition examples.

1 Introduction

Recent advances in VLSI technology and design has made possible the realization of low cost Application Specific Array Processors (ASAPs) for computationally intensive image and signal processing algorithms. A well known approach to VLSI implementation of a signal processing algorithm is to design a *systolic* array for that algorithm. A systolic array is an ASAP made up of regularly-connected processing elements, all of which are connected locally and exploit the inherently regular nature of signal processing algorithms in order to achieve the high throughput demands of the tasks.

Considerable effort has been expended by researchers to find procedures to map algorithms onto a systolic array [1,2,3]. These methods aim to systematically map a given algorithm into a high-level hardware processor array form. Very often the array that is obtained by these mapping procedures is suited only for a particular size of the problem. An array designed for that size only could be very inefficient or even unusable if the target problem size were bigger. It is critical that the design system should incorporate a technique for *partitioning* a larger problem as well as provide extra hardware devices that can handle this partitioning.

In section 2, we review the main issues involved in partitioning systolic algorithms, along with a brief description of the two main categories of partitioning methods. In section 3, we discuss a scheduling method based on modification of the locality constraints of partitioned Dependence Graphs, as well as a scheduling order for LPGS scheme when there is a constraint on the size of the buffers that are needed for partitioning. The benefits of the proposed procedures are illustrated with the help of matrix multiplication and QR decomposition algorithms in Section 4.

2 Mapping & Partitioning issues

General partitioning methods are often derived from the mapping procedures for full-size arrays. Most mapping procedures are only applicable to a class of algorithms that can be represented by nested loops with uniform dependencies. The algorithms are written in the form of for-loops and in Single-Assignment form; i.e. each variable is assigned a value only one time during the algorithm, no variable is overwritten. The algorithm is then represented by an indexed dependence graph (DG) in an Index space. The DG contains nodes at various points in the Index space representing all the computations that are indexed by that index point. The edges represent the data dependencies between computations at various index points. This DG form is useful in removing unwanted data broadcasts by allowing dependencies between adjacent nodes only. This spatial locality preserves the local nature of inter-processor connections characteristic of a systolic array.

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Comparison of Spatially Smoothed Weighted Subspace Methods*

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Abstract

In this paper, the performance of spatially smoothed weighted MUSIC, and weighted state space methods/ESPRIT are analyzed, and compared for the direction of arrival (DOA) estimation problem. In particular, a key outcome of the analysis is the observation that when a forward-backward spatially smoothed approach is used, optimally weighted MUSIC and optimally weighted state space methods/ESPRIT have identical asymptotic behavior. Also it is shown that the forward-backward covariance matrix is better conditioned than the forward only covariance matrix providing additional justification for the use of the forward-backward approach. The analysis is shown to be extendable to the time series frequency estimation problem, thereby providing a unified framework for dealing with DOA estimation and time series frequency estimation.

1 INTRODUCTION

In this paper, the effect of using a Forward-Backward spatially Smoothed (FBS) covariance matrix on the statistical performance of weighted eigen-based state space based methods or ESPRIT and weighted MUSIC is analyzed. In recent years, a statistical evaluation of subspace based methods has been conducted by a number of researchers with particular attention being paid to the case where the estimate of the covariance matrix was obtained by a straight forward averaging of the outer product of the snapshots. It was shown that MUSIC performs better than eigen-based state space methods with the difference becoming significant as the length of the array increases. However, state space methods are computationally more attractive and there is incentive to seek ways to improve the statistical properties. It is shown that this deficiency can be overcome by using a spatially smoothed forward-backward covariance matrix and proper weighting. A key result of the analysis is that optimally weighted MUSIC and optimally weighted State Space Methods/ESPRIT have identical asymptotic performance. Amongst the covariance estimators, it is shown that combining spatial smoothing with the forward-backward approach is more effective than using forward spatial smoothing alone. Another important attribute of the FBS approach for the problem of narrowband DOA estimation using an ULA is that it is naturally related to the time series frequency estimation problem, i.e. sinusoids in noise problem. This paper capitalizes on this relationship and the analysis framework [1,2] to establish an unified framework for these problems.

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2 PROBLEM FORMULATION

The output vector of a ULA of L_1 sensors is given by

$$Y(n) = \left[\sum_{i=1}^{M} p_i(n) \ , \ \ldots , \sum_{i=1}^{M} p_i(n) \ e^{j(L_1-1)\omega_i} \right]^T + N(n)$$

where $\omega_i = \frac{2\pi d}{\lambda} \sin\theta_i$, d being the separation between sensors, λ the wavelength of the incident signal, and θ_i the DOA. Due to the simple mapping between ω_i and θ_i, for simplicity, ω_i will also be often referred to as the DOA.

2.1 FBS Covariance Matrix Estimate

Exploiting the structure in the ULA, the general FBS estimate is given by

$$\hat{R}_{fL_1} = \frac{\hat{R}_{f_1} + J\hat{R}^*_{f_1}J}{2} \quad \textit{where} \ J = \begin{bmatrix} 0 & 0 & 1 \\ \vdots & \vdots & \vdots \\ 0 \ldots & 1 & \ldots 0 \\ \vdots & \vdots & \vdots \\ 1 & 0 & 0 \end{bmatrix}, \qquad (1)$$

Floating Point Arithmetic and Digital Filters

Bhaskar D. Rao, *Member, IEEE*

***Abstract*—In this paper, the roundoff noise properties of floating point digital filters are examined. To make the analysis tractable, a high level model to deal with the errors in the inner product operation is developed. This model establishes a broad connection between coefficient sensitivity and roundoff noise. Along with the model, an efficient procedure to keep track of the addition scheme used in the inner product, and to compute the statistics of the errors is introduced. A systematic procedure based on the model is then developed to derive general expressions for the roundoff noise of FIR, direct form IIR, and state-space filters. The expressions in the context of state space filters are explored in some detail. Optimality issues are considered, and it is shown that when double precision accumulation is used, the optimal filters are similar in nature to those derived in the context of fixed point arithmetic with the essential difference that they also do depend on the spectrum of the input signal. Optimality with respect to addition schemes, and second-order filters are also examined in some detail.**

I. Introduction

FLOATING point arithmetic is widely used in signal processing computations. More recently, with the increasing availability of floating point capability in signal processing chips, insights into algorithms employing floating point arithmetic is of increasing interest. This paper considers the problem of digital filter implementation, and examines the effect of floating point arithmetic on them. The related problem of using fixed point arithmetic and their effect on digital filters has been extensively studied and is reasonably well understood [1]-[5]. The analysis of floating point digital filters has also been studied by researchers [6]-[14]. Unfortunately, the expressions turn out to be cumbersome and not as insightful. Part of the reason for the difficulty lies in the model for finite precision floating point arithmetic. Unlike the case of fixed point arithmetic, where the roundoff errors could be modeled as independent white noise sequences independent of the input signal, the roundoff errors in floating point arithmetic are correlated with the input signal complicating the analysis [7], [6]. Also, unlike the fixed point case, the order of the computations has an effect on the roundoff error. Recently, in [15], it was shown that use of the factored state variable description can greatly enhance the tractability of the analysis. It was used to derive expressions for the roundoff noise of arbitrary filter structures, and to provide a quantitative comparison of fixed and floating point implementations. More recently, it has been shown that the tractability of the analysis can be enhanced by connecting roundoff analysis to coefficient sensitivity analysis [26], [27], [17]. This paper reexamines the issue of output roundoff noise of floating point digital filters with a view towards generalizing the existing results, and providing a more tractable and hopefully insightful approach for dealing with floating point errors.

The outline of this paper is as follows. To enhance the tractability of the analysis, in Section II, a high level model for the inner product, an operation basic to digital filters, is developed. The model indicates a broad relationship between roundoff noise and coefficient sensitivity. Along with the model, an efficient procedure to keep track of the addition scheme used in the inner product, and to compute the statistics of the error is introduced. A systematic approach based on this model is then developed to derive expressions for the output roundoff noise of different filter structures (Section IV). The FIR and IIR filter structures are examined in Section III. The results serve to illustrate the generality and simplicity of the approach. The potential of this approach is further demonstrated by analyzing the more complicated state-space digital filters in Section V. An exact expression for the variance of the output roundoff noise is derived when the input to the filter is a zero-mean wide-sense stationary random process. It explicitly indicates the effect of the filter parameters, the statistics of the input signal, and the addition scheme used in the computation on the roundoff noise. Various properties and optimality issues relating to state space filters are considered in Section VI. For instance, when double precision accumulation is used, it is shown that the optimal filter structures are similar to those obtained when using fixed point arithmetic. Issues related to addition schemes, and second-order structures are also examined. Some of these results were first presented in [16], [17].

II. Inner Products and Floating Point Arithmetic

A. Basic Error Model

First we discuss the errors that arise when floating point arithmetic is used, and develop a model to cope with these errors in the context of digital filters. Throughout this paper it will be assumed that floating point numbers are stored in the form (sign) $\cdot\ \mu \cdot 2^{\nu}$, where μ and ν have a fixed number of bits. Also it will be assumed that round-

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Model Based Processing of Signals: A State Space Approach

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This paper is a tutorial on linear, state space, model-based methods for certain nonlinear estimation problems commonly encountered in signal and data analysis. A prototypical problem that is studied is that of estimating the frequencies of multiple, superimposed sinusoids from a short record of noise-corrupted data. The approach expounded however, is applicable to a vast range of nonlinear signal analysis problems, and applications in direction finding and damped sinusoid retrieval are dealt with in some detail. The benefits that result from using a state space description of the signal are highlighted in this paper. It is shown that state space models provide an elegant tool for exposing the structure present in the problem. The approach also allows for robust parameterization of the model with respect to finite precision errors. The robustness of the parameter set is complemented by the availability of numerically robust tools to estimate the parameters. The resulting algorithms are compatible with multiprocessor implementations.

I. Introduction

The problem of estimating closely spaced frequencies of multiple, superimposed sinusoids from noisy measurements of the time-series or its covariance lags, is prototypical of a class of nonlinear parameter estimation problems that has a vast range of signal processing applications. Some other problems that belong in this class are the problem of estimating the damping factors and frequencies of damped sinusoids and exponentials from a linear combination of the same, the identification of a linear, time-invariant, rational system from impulse response or input–output data, model-based power spectrum estimation for a stochastic process, the problem of separating a small number of overlapping echoes of an unknown signal, time delay estimation, and the problem of narrow-band source bearing estimation using a multisensor array. These and closely related problems appear in many different fields within electrical engineering, and have been addressed by researchers in diverse disciplines, such as sonar, radio astronomy, interference spectroscopy, seismic data processing, and NMR spectroscopic imaging.

In recent years there has been a great deal of attention paid to model-based eigendecomposition methods for these problems. The use of linear models for the signal to be processed, can convert a nonlinear estimation problem into the simpler problem of estimating the parameters of the linear model [1]. The nonlinearity is postponed to the second step, which in all model-based methods is the step where the desired information (frequencies, damping factors, directions of arrival) is extracted from the estimated model parameters. Both steps are crucial to the overall success of a model-based method, and ill-conditioning at either step can adversely effect the method's performance. The reliability of the first step depends on the model-parameter estimation scheme, and that of the second step on the sensitivity of the desired information to the model parameters. Often in signal processing, a polynomial realization of the linear model has been assumed and used. A state space representation of the linear model is a viable alternative, and is sometimes desirable for these problems. State space representation provides flexibility in the model parameterization, and can make both steps of the model based procedure reliable.

This paper reviews the state of the art in state space model based signal processing, and discusses the advantages of adopting a state space approach. While the presentation deals mainly with the representative problem of sinusoid retrieval, the issues and approach are applicable to other related problems including the ones listed above. In each of these problems, an invariance structure inherent to the data is made explicit by the state space approach, which can be used to extract the desired information regarding the signal. In addition, the state space approach is shown to provide a degree of flexibility in model parameterization, which if properly exploited can greatly enhance performance. In contrast to the more classical polynomial or transfer function parameterization which requires polynomial rooting to get to the final quantities of interest, state space parameterization allows the use of numerically robust techniques such as eigendecomposition of well-conditioned matrices to extract the desired quantities. Also, it will be seen that

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Efficient scheduling methods for partitioned systolic algorithms *

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Abstract

Various methods for mapping signal processing algorithms into systolic arrays have been developed in the past few years. In this paper, efficient scheduling techniques are developed for the partitioning problem, i.e. problems with size that do not match the array size. In particular, scheduling for the Locally Parallel-Globally Sequential (LPGS) technique and the Locally Sequential-Globally Parallel (LSGP) technique are developed. The scheduling procedure developed exploits the fact that after LPGS and LSGP partitioning, the locality constraints get modified allowing for more flexibility and the use of inter-processor connections that were not available before. The new structure allows us to develop a flexible scheduling order for LPGS that is usefu n evaluating a trade-off between execution time and size of partitioning buffers. The benefits o, the scheduling techniques are illustrated with the help of matrix multiplication and Least Squares examples.

1 Introduction

Recent advances in VLSI technology and design has made possible the realization of low cost Application Specific Array Processors (ASAPs) for computationally intensive image and signal processing algorithms. In real-time signal processing applications, ASAPs offer an attractive alternative to general purpose computing machines. A well known approach to implementation of an algorithm is to design a *systolic* array for that algorithm. A systolic array is an ASAP made up of regularly-connected processing elements, all of which are connected locally and exploit the inherently regular nature of signal processing algorithms in order to achieve the high throughput demands of the tasks.

Considerable effort has been expended by researchers in the past few years to find systematic procedures to map algorithms onto a systolic array [1,2,3]. Although these tools provide a framework for synthesis and analysis of systolic arrays, there are some limitations when using them in practical design scenarios. Most DSP algorithms involve large data vectors or matrices for which the systolic array would require a large number of Processing Elements (PEs). Using a large number of PEs poses system level and VLSI problems. Current VLSI technology still does not allow for implementing a really large number of PEs on a single chip, no matter how simple the processor structure. Using a number of processor-chips leads to problems with data I/O. Integrated Circuits have a limited number of pins and therefore crossing chip boundaries becomes a bottleneck [4].

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Scheduling parallel implementations of partitioned orthogonal transformations

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ABSTRACT

Orthogonal matrix transformations form an important part of matrix-based signal processing applications. Systolic arrays for computing these algorithms have been developed and the size of these arrays usually depends directly on the size of the problem. For large matrix sizes, implementing large numbers of processors in hardware is not physically feasible. In this paper, we examine two popular orthogonal transformations, Givens Rotations and Householder Transformations (HT), from the viewpoint of realizing a fixed-size parallel processor array that can handle large data matrices. An efficient scheduling procedure is used to compute the HT on a systolic type array, its performance is compared with that of an array designed for computing the Givens method. An important conclusion resulting from the comparison is that the performance of the HT array is superior to that for the Givens method when the matrices are larger compared to the array size.

1. INTRODUCTION

There are several applications in signal processing where orthogonal triangularization procedures are very helpful. For example, methods like Givens Rotations and Householder Transformations (HT)[1] are used for computing QR Decomposition (QRD), Least-Squares filtering, eigendecomposition, SVD, etc.

The method of Givens Rotations has been a popular choice in computing the QRD. This is a computationally simple method that has been found to be amenable to implementation on special-purpose parallel computing machines like multiprocessor arrays and systolic structures. Several systolic arrays can be found in literature[2] which use Givens Rotations in order to compute matrix operations like QRD, etc. The computations required by the Givens method are spatially local by nature and therefore an attractive choice for implementation on a systolic array.

Unlike the Givens method which annihilates one element of a matrix/vector at a time, the HT approach is a flexible way by which several elements of a matrix or a vector can be annihilated at one time. From the standpoint of numerical performance (finite precision effects), the HT is a superior method when compared to the Givens method[3]. Another reason that makes the HT the more attractive method to use is its computational complexity. For example, the numbers of flops required to compute the QRD of a $p \times n$ matrix using Givens Method is $2n^2(p - n/3)$ while

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A New Iterative Weighted Norm Minimization Algorithm and its Applications*

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Abstract

A general class of linear inverse problems in which the solutions are sparse and localized are considered in this paper. We propose an iterative weighted minimum norm algorithm for finding such constrained solutions. A unique feature of this algorithm is that it is a nonparametric approach that finds sparse and localized solutions without prior information on the constraints. Each step of the iterative procedure consists of solving a weighted least squares problem wherein the weighting matrix is determined by the solution from the previous iteration. Some properties of the algorithm along with its applications to problems in direction of arrival (DOA) and spectrum estimation are presented.

1 Introduction

An important problem in signal processing is signal characterization and reconstruction from a limited linearly mapped set of measurements. The general problem is underdetermined and characterized by having an infinite set of possible solutions. The nonuniqueness is resolved by further constraining the problem with prior information [1]. Here we discuss a nonparametric approach which has potential applications in spectral estimation, direction-of arrival (DOA) estimation, and other signal analysis problems where the solution is assumed to be sparse and localized. No prior information on the number and nature of the localizations in the solution space is assumed. For example, in the narrowband DOA estimation application, given sensor measurements the objective is to reconstruct a spatial spectrum that has energy localized in certain spatial regions [2]. The advantage of the nonparametric approach in this context is that it can handle sources with small finite extent in addition to point sources. The algorithm was first developed for the biomagnetic neuroimaging inverse problem [3,4]. This paper further explores the algorithm and its potential applications in signal processing.

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2 Background

The general discrete underdetermined linear inverse problem consists of finding a solution to the system

$$Ax = b, \tag{1}$$

where A is $m \times n$, $m < n$, and has rank m. The number of solutions is infinite and the solution vectors are given by $x = x_{mn} + v$, where v is a vector in the null space of A and

$$x_{mn} = A^{+}b, \tag{2}$$

A^{+} denotes the Moore-Penrose inverse [5]. The vector x_{mn}, called the minimum norm or the minimum energy solution lies entirely in the range space of A^H and has the unique property of being the vector of smallest 2-norm satisfying (1).

In this paper, solutions to (1) that are sparse, localized, and contiguous are sought. The need for a sparse x can be motivated by considering the problem of sinusoid frequency estimation. The measurement vector b consists of m samples of data which are a sum of p complex exponentials ($p < m$). The columns of A are of the form $[1, e^{-j\omega}, e^{-j2\omega}, ..., e^{-j(m-1)\omega}]^T$. The n columns of A are formed by choosing ω from the range $(-\pi, \pi)$. The choice of the values of ω is rather arbitrary and may reflect prior knowledge. If $m = n$, and ω is is chosen by uniformly sampling the frequency axis,

Weighted Subspace Methods and Spatial Smoothing: Analysis and Comparison

Bhaskar D. Rao, *Senior Member, IEEE*, and K. V. S. Hari

Abstract—In this paper, the effect of using a spatially smoothed forward-backward covariance matrix on the performance of weighted eigen-based state space methods/ESPRIT, and weighted MUSIC for the direction-of-arrival (DOA) estimation is analyzed. Expressions for the mean-squared error in the estimates of the signal zeros and the DOA estimates, along with some general properties of the estimates and optimal weighting matrices are derived. A key result of the analysis is that optimally weighted MUSIC and weighted state space methods/ESPRIT have identical asymptotic performance. It is also shown that by properly choosing the number of subarrays, the performance of unweighted state space methods can be significantly improved. Then it is shown that the mean-squared error in the DOA estimates obtained using subspace based methods is independent of the exact distribution of the source amplitudes. This results in an unified framework for dealing with DOA estimation using a uniformly spaced linear sensor array (ULA), and the time series frequency estimation problem. The resulting analysis of the time series case is shown to be more accurate than previous results.

I. Introduction

In this paper, weighted subspace based methods for estimating the direction of arrival (DOA) of plane waves in noise using a uniformly spaced linear array (ULA) are considered. In particular, the effect of using a spatially smoothed forward-backward covariance matrix on the statistical performance of weighted eigen-based state space based methods [1] or ESPRIT [2] and weighted MUSIC is analyzed. An examination of unweighted MUSIC method under these conditions has recently been studied in [3]-[6], and hence in this paper there is greater emphasis on the weighted state space methods/ESPRIT. ESPRIT was developed for general arrays which possess a displacement invariance [2] and reduces to a state space method for the ULA case [1], [7], [8]. However, the state space methods were developed first and deal extensively with the ULA case, the problem of interest here [1], [8]-[10]. In particular, the use of spatial smoothing and the forward-backward approach in the manner developed here were first discussed in the context of state space methods. Hence, the use of the term state space methods appears more appropriate and will be used in this paper.

In recent years, a statistical evaluation of subspace based methods has been conducted by a number of researchers [3], [4], [8], [11]-[19]. Particular attention has been paid to the case where the estimate of the covariance matrix was obtained by a straight forward averaging of the outer product of the snapshots. In [15], [17], [8], [6], it was shown that MUSIC performs better than eigen-based state space methods with the difference becoming significant as the length of the array increases. However, state space methods are computationally more attractive and there is incentive to seek ways to improve the statistical properties. Weighted eigen-based state space methods and weighted MUSIC are analyzed, and it is shown that this deficiency can be overcome by using a spatially smoothed forward-backward covariance matrix and proper weighting. These results are consistent with the past successful uses of the forward-backward approach, first for autoregressive parameter estimation [20], and later for the problem of sinusoid frequency estimation [21], [9], and for DOA estimation of coherent sources [22]-[24].

Another important attribûte of the forward-backward smoothing approach is that it enables a natural link between the problem of narrow-band DOA estimation using an ULA, and the time series frequency estimation problem, i.e., sinusoids in noise problem. The natural commonality between these problems is well known, however, unfortunately the analysis of these problems has been treated separately. This paper capitalizes on the analysis framework developed in [3], [5], and establishes an unified framework for these problems.

The outline of the paper is as follows. Section II provides some background information. The various covariance matrix estimators are discussed and the framework for analysis is outlined. Amongst the covariance estimators, it is shown that combining spatial smoothing with the forward-backward approach is more effective than using forward spatial smoothing alone. The statistics relevant to the analysis are derived under the assumption that the source amplitudes and the noise are independent and are circularly Gaussian random vectors.

In Section III, an expression for the mean-squared error (MSE) in the DOA estimates obtained using eigen-based state space methods is derived. Some interesting proper-

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A methodology for fast scheduling of partitioned systolic algorithms *

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Abstract

Various methods for mapping signal processing algorithms into systolic arrays have been developed in the past few years. In this paper, efficient scheduling techniques are developed for the partitioning problem, i.e. problems with size that do not match the array size. In particular, scheduling for the Locally Parallel-Globally Sequential (LPGS) technique and the Locally Sequential-Globally Parallel (LSGP) technique are developed. The scheduling procedure developed exploits the fact that after LPGS and LSGP partitioning, the locality constraints are less stringent allowing for more flexibility in the choice of algorithms and inter-processor communication. A flexible scheduling order is developed that is useful in evaluating the trade-off between execution time and size of storage buffers. The benefits of the scheduling techniques are illustrated with the help of matrix multiplication and least squares examples.

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An index prediction based Vector Quantization scheme

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Abstract

In this paper, a new VQ coder is developed that exploits inter-vector correlation in a simple and inexpensive manner. A key and important difference between the suggested method and existing methods is that it involves prediction of the index that is assigned to a data vector as opposed to the prediction of data vectors. Success of the method depends on properly assigning the indices to the codebook vectors, and for this purpose three schemes are suggested. This methodology results in a substantial reduction in the required Bit-rate per pixel without any additional loss in image quality.

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